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BOOK REVIEW

Difference Equations and Inequalities : Methods and Applications
by Ravi P Agarwal
Marcel Dekker : New York-Basel Hong-Kong, 1992
xiii + 777 pages, illustrated ; price : 150.00 (bound) ; ISBN 0-8247-8676-9

This book comprises 12 chapters dealing with the solutions of difference equations and their applications to physical problems.

The book gives a clear picture on the solution of linear and non-linear difference equations. It explains in detail, the discrete versions of Rolle's theorem, the mean value theorem and Kneser's theorem. Taylor's formula and L'Hospital's theorem have been discussed with illustrations and physical applications. Some important topics on Duffing's, Vander Pol's and Hill's equations with discrete interpretations have also been explained.

The stability and oscillatory properties of solutions of difference equations have been fairly discussed and explained. This book reviews the theory of difference inequalities. The unified treatment of boundary value problems concerning the solution of linear and non-linear difference equations has been explained in detail.

Chapter I deals with some preliminaries.

Chapter II contains Lagrange's and Green's identities in a very lucid way. The classical method after Bernoulli and some related results like Poincare's and Perron's theorem have also been discussed.

In Chapter III, the author discusses several difference equations starting from Clairaut's, Euler's and Riccati's forms. He develops a discrete approximation of the linear differential equations with constant co-efficients. Using this method, Duffing's difference equation can be solved explicitly. The author has shown that the Weierstrass' elliptic differential equations can be discretized and the solutions of the final equation is in accord with the expected results from the elliptic functions. The most interesting things are the discretization like of Wave equations like Fitz Hugh Nagumo's equation, Korteweg-de Vries' equation and Modified KdV equation.

In Chapter IV, the author has discussed Grownwall's inequalities and Opial's type inequalities in detail.

In Chatper V, he has shown the qualitative properties of solutions of difference systems. This concept is of various computational importance. The author has given an idea of practical stability of the solutions, which goes beyond the classical Lyapunov stability theory and shown some applications in numerical analysis.

In Chapter VI, the author has discussed the qualitative properties of solutions of higher order difference equations. The systematic treatment of oscillation and non-oscillation theory of difference equations has heen inserted.

In Chapter VII, the author provides the necessary and sufficient conditions for the existence and uniqueness of the solutions of linear boundary value problems. The solution of some problems based on Green's matrices, has been represented. The minimal solution of the difference equations is of importance in numerical analysis. In this connection, the author has discussed the algorithms of Miller and Olver.

In Chapter VIII, the author has defined the generalized normed spaces and state of two fixed point theorems. He also proved the existence and uniqueness of the solutions of the non-linear boundary value problems. The most interesting thing is that he provides *a priori* sufficient conditions which ensure the convergence of Picard's iterative scheme to its unique solution. The computational aspects of the Picard's scheme on floating point system has been discussed. The author has also introduced various partial orderings in the space $B(K_1, K_r)$ of the Picard's scheme to its solutions. The convergence of the Newton's and approximate Newton's methods for non-linear problems are discussed in detail.

In Chapter IX, the author has discussed disconjugacy, right disfocality, eventual disconjugacy and eventual right disfocality for the linear homogeneous difference equations. He has discussed Polya's factorization and relationship between D-Markov, D-Fekete and D-Descartes systems. The author has given some special attention to discrete interpolating polynomials and explicit representation of Green's functions for several higher-order boundary value problems.

In Chapter X, the author provides verifiable sets of necessary and sufficient conditions so that each of these boundary value problems has atleast one solution. He has demonstrated some easier tests for the local existence and uniqueness of the solutions of higher order boundary value problems.

Chapter XI consists of special type of boundary value problems that give an idea on eigenvalues and eigenfunctions, orthogonality and finite Fourier series. The author has derived Wirtinger's and Optical inequalities. The Strum-Liouville problems and other related inequalities are discussed in detail.

In Chapter XII, the author has established the discrete analog of Riemann's function. This is important in the study of linear Gronwall type inequalities. He has given an estimate of the Riemann's function. The inequalities involving higher order differences in two independent variables are also discussed. Taylor's formula with two independent variables is

also studied. Attention has been paid to the multidimensional linear and nonlinear discrete inequalities. The author has developed Opial's and Wirtinger's type inequalities with two independent variables.

The exhaustive compilation and analytic representation of the various mathematical problems are indeed useful.

The difference equations is a mathematical tool which can be employed for the interpretation of physical problems, statistical problems, stochastic time series, combinatorial analysis, number theory, geometry, electrical networks, quanta in radiation, etc. Difference equations have wider applications in the modifications of computer software fabrication, since the differential equations are solved by using approximate difference equation formulations.

This book is essential for the enrichment of knowledge in mathematics, physics and statistics. The comprehensive compilation of the book is useful for researchers of natural philosophy.

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